Wed April 6 Sound Simulation

Why Dublin is the biggest city in Ireland? Because it keeps doubling, doubling and doubling.

Start with several demos:

There are two ways to generate sounds:

1. Record clips. Original way. We place the microphone and the recorder in a room that has no echo, and record all kinds of sound. Most sounds in games are done in this way, although it is easy to do real time simulation. But the drawback is that we can't change the quality of the sound, we can't change it to other sounds. And that's why we need physical simulation.

2. Simulate. Sound is produced by vibration, which is very fast deformation. Objects vibrating with high frequency. For example, an iPhone dropped onto a table will produce some sound, it is a mix of all frequency. The same thing happens to the table. Both objects are vibrating when we hear or record. Also, air creates shock wave. We use wave equations to simulate sound.

The equations are produced by rigid body colliding model. metallic objects are easier to simulate. It can be done in real time, is cheap and easy to do, and can be put in games.

There is a SIGGRAPH paper in 2002 that simulates sound. Although there are ways to generate better sounds, only the equations in that paper is discussed in class, which is called BEM/PAT:

\[ Mu''+Du'+Ku=f(t) \]

Where \( u \) is the deformation of the object, \( M \) is the mass matrix, \( D \) is damping, and \( K \) is stiffness.

\[ Kx = \lambda \ Mx \]

Where \( \lambda \) is eigenvalue matrix.

Since \( K \) and \( M \) is known, we can use put this to the solver and get the \( \lambda \).

\[ 0<\lambda \ 1<\lambda \ 2<... \]

where

\[ \lambda_i = \omega_i^2 \]

and

\[ \omega_i = 2\pi \psi_i \]

Where \( \psi \) is the frequency

Using model reduction, we can create another matrix \( U = [\psi 1, \psi 2, ..., \psi 10] \)
that \( u = Uq \)
where \( q = [q_1, q_2, \ldots, q_{10}]^T \) which is another variable used to replace \( u \).

\[
    u(t) = Uq(t) = \sum_{i=1}^{10} \psi_i \cdot q_i(t)
\]

Then we have:

\[
    MUq'' + DUq' + KUq = f(t)
\]

becomes:

\[
    U^T MUq'' + U^T DUq' + U^T KUq = U^T f(t)
\]

where

\[
    U^T MU = I
\]

because

\[
    \psi_i^T M \psi_j = \begin{cases} 0 & (i \neq j) \\ 1 & (i = j) \end{cases}
\]

and because:

\[
    \psi_i^T K \psi_j = \psi_i^T \lambda_j \psi_j
\]

so:

\[
    U^T K U = \Lambda
\]

where

\[
    \Lambda = \begin{bmatrix} 
    \lambda_1 & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & \lambda_{10} 
    \end{bmatrix}
\]

Assume

\[
    D = \alpha M + \beta K
\]

then we have:

\[
    q'' + (\alpha I + \beta \Lambda)q' + \Lambda q = U^T f(t)
\]

And since

\[
    q = [q_1, q_2, \ldots, q_{10}]^T
\]

For each \( q \):

\[
    q'' + (\alpha I + \beta \Lambda)q' + \Lambda q_i = \psi_i^T f(t)
\]

Solve this equation, we get something like:

\[
    q_i(t) = e^{-\beta t} \sin(\omega_i t)
\]

where:

\[
    \omega_i^2 = \lambda_i
\]

which looks like:
Then, we add all signals together and send to the loud speaker.