Physically Based Simulation

Examples
Particle Systems
Numerical Integration
Cloth Simulation
[Angel Ch. 11.2-11.6]
Physics in Computer Graphics

- Very common
- Computer Animation, Modeling (computational mechanics)
- Rendering (computational optics)
Physics in Computer Animation

- Fluids
- Smoke
- Deformable strands (rods)
- Cloth
- Solid 3D deformable objects .... and many more!
Fluids

Enright, Marschner, Fedkiw, SIGGRAPH 2002
Fluids and Rigid Bodies

[Carlson, Mucha, Turk, SIGGRAPH 2004]
Fluids with Deformable Solid Coupling

[Robinson-Mosher, Shinar, Gretarsson, Su, Fedkiw, SIGGRAPH 2008]

Two-way Coupling of Fluids to Rigid and Deformable Solids and Shells

Avi Robinson-Mosher  
Tamar Shinar  
Jon Gretarsson  
Jonathan Su  
Ronald Fedkiw
Deformations

Vertices: 45882
Triangles: 105788

[Barbic and James, SIGGRAPH 2005]
Cloth

Source: ACM SIGGRAPH
Cloth (Robustness)

[Bridson, Fedkiw, Anderson, ACM SIGGRAPH 2002]
Simulating Large Models

[Doug James, PhD Thesis, UBC, 2001]
Sound Simulation (Acoustics)

Modal renderer

[James, Barbic, Pai, SIGGRAPH 2006]
Multibody Dynamics

Figure 1: *Avalanche:* 300 rocks tumble down a mountainside.
Multibody Dynamics + Self-collision Detection

[Barbic and James, SIGGRAPH 2010]
Physics in Games

Real-Time Deformation and Fracture in a Game Environment

Eric Parker
Pixelux Entertainment

James O’Brien
U.C. Berkeley

Video Edited by Sebastian Burke

From the proceedings of SCA 2009, New Orleans

[Parker and James, Symposium on Computer Animation 2009]
Haptic Interfaces

- hap·tic (ˈhap-tik) adj.
  Of or relating to the sense of touch; tactile.
Surgical Simulation

[James and Pai, SIGGRAPH 2002]
Offline Physics

- Special effects (film, commercials)
- Large models: millions of particles / tetrahedra / triangles
- Use computationally expensive rendering (global illumination)
- Impressive results
- Many seconds of computation time per frame
Real-time Physics

• Interactive systems:
  computer games
  virtual medicine (surgical simulation)

• Must be fast (30 fps, preferably 60 fps for games)
  Only a small fraction of CPU time devoted to physics!

• Has to be stable, regardless of user input
Particle System

- Basic physical system in computer graphics
- We have $N$ particles
- They interact with some forces
- Fire, Smoke, Cloth, ...
- Very popular for its simplicity

[William Reeves, SIGGRAPH 1983]
Newton’s Laws

• Newton’s 2nd law:

\[ \vec{F} = m \vec{a} \]

• Gives acceleration, given the force and mass

• Newton’s 3rd law: If object A exerts a force \( F \) on object B, then object B is at the same time exerting force \(-F\) on A.
Case Study: Mass-spring Systems

- Mass particles connected by elastic springs
- One dimensional: rope, chain
- Two dimensional: cloth, shells
- Three dimensional: soft bodies

Source: Matthias Mueller, SIGGRAPH
Single spring

- Obeys the **Hook’s law**: 
  \[ F = k (x - x_0) \]
- \( x_0 \) = rest length
- \( k \) = spring elasticity (stiffness)
- For \( x < x_0 \), spring wants to extend
- For \( x > x_0 \), spring wants to contract
Hook’s law in 3D

• Assume A and B two mass points connected with a spring.
• Let L be the vector pointing from B to A
• Let R be the spring rest length
• Then, the elastic force exerted on A is:

\[ \vec{F} = -k_{Hook} (|\vec{L}| - R) \frac{\vec{L}}{|\vec{L}|} \]
Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

\[ \vec{F} = -k_d \vec{v} \]

- \( k_d \) = damping coefficient
- \( k_d \) different than \( k_{Hook} \)!!
A network of springs

- Every mass point connected to some other points by springs

- Springs exert forces on mass points
  - Hook’s force
  - Damping force

- Other forces
  - External force field
    - Gravity
    - Electrical or magnetic force field
  - Collision force
Network organization is critical

• For stability, must organize the network of springs in some clever way

Basic network  Stable network  Network out of control
Physics equation:
\[ x' = f(x,t) \]

\( x = x(t) \) is particle trajectory

Source: Andy Witkin, SIGGRAPH
Euler Integration

\[ x(t + \Delta t) = x(t) + \Delta t \, f(x(t)) \]

Simple, but inaccurate.

Unstable with large timesteps.

Source: Andy Witkin, SIGGRAPH
Inaccuracies with explicit Euler

Gain energy

“Blow-up”

Source: Andy Witkin, SIGGRAPH
Midpoint Method

Improves stability

1. Compute Euler step
   \[ \Delta x = \Delta t \, f(x, t) \]

2. Evaluate \( f \) at the midpoint
   \[ f_{\text{mid}} = f((x+\Delta x)/2, (t+\Delta t)/2) \]

3. Take a step using the midpoint value
   \[ x(t+ \Delta t) = x(t) + \Delta t \, f_{\text{mid}} \]

Source: Andy Witkin, SIGGRAPH
Many more methods

• Runge-Kutta (4th order and higher orders)

• Implicit methods
  – sometimes unconditionally stable
  – very popular (e.g., cloth simulations)
  – a lot of damping with large timesteps

• Symplectic methods
  – exactly preserve energy, angular momentum and/or other physical quantities
  – Symplectic Euler
Cloth Simulation

- Cloth Forces
  - Stretch
  - Shear
  - Bend

- Many methods are a more advanced version of a mass-spring system

- Derivatives of Forces
  - necessary for stability

[Baraff and Witkin, SIGGRAPH 1998]
Challenges

• Complex Formulas
• Large Matrices
• Stability
• Collapsing triangles
• Self-collision detection

[Govindaraju et al. 2005]
Self-collisions: definition

Deformable model is self-colliding iff there exist non-neighboring intersecting triangles.
Bounding volume hierarchies

[Hubbard 1995]
[Gottschalk et al. 1996]
[van den Bergen 1997]
[Bridson et al. 2002]
[Teschner et al. 2002]
[Govindaraju et al. 2005]
Bounding volume hierarchy

root

root
Bounding volume hierarchy
Real-time cloth simulation

<table>
<thead>
<tr>
<th>Model</th>
<th>Triangles</th>
<th>FPS</th>
<th>% Forces + Stiffness Matrix</th>
<th>% Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curtain</td>
<td>2400</td>
<td>25</td>
<td>67</td>
<td>33</td>
</tr>
</tbody>
</table>

Source: Andy Pierce
Multithreading implementation

Force and Stiffness Matrix Computation

Milliseconds

Number of Threads

Source: Andy Pierce
Summary

• Examples of physically based simulation
• Particle Systems
• Numerical Integration
• Cloth Simulation