Rasterization

Rasterization (scan conversion)

- Final step in pipeline: rasterization
- From screen coordinates (float) to pixels (int)
- Writing pixels into frame buffer
- Separate buffers:
  - depth (z-buffer),
  - display (frame buffer),
  - shadows (stencil buffer),
  - blending (accumulation buffer)

Rasterizing a line

Digital Differential Analyzer (DDA)

- Represent line as
  \[ y = mx + b \]
  where \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \]
- Then, if \( \Delta x = 1 \) pixel, we have \( \Delta y = m \Delta x = m \)

Digital Differential Analyzer

- Assume write_pixel(int x, int y, int value)
  for (i = x1; i <= x2; i++)
  {
    y += m;
    write_pixel(i, round(y), color);
  }
- Problems:
  - Requires floating point addition
  - Missing pixels with steep slopes: slope restriction needed

Digital Differential Analyzer (DDA)

- Assume \( 0 \leq m \leq 1 \)
- Exploit symmetry
- Distinguish special cases

But still requires floating point additions!
Bresenham's Algorithm I

- Eliminate floating point addition from DDA
- Assume again 0 ≤ m ≤ 1
- Assume pixel centers halfway between integers

\[
\begin{align*}
&j + \frac{3}{2} \\
&j + \frac{1}{2} \\
&j - \frac{1}{2} \\
&i + \frac{1}{2} \\
&i - \frac{1}{2}
\end{align*}
\]

Bresenham's Algorithm II

- Decision variable a − b
  - If a − b > 0 choose lower pixel
  - If a − b ≤ 0 choose higher pixel
- Goal: avoid explicit computation of a − b
- Step 1: re-scale \( d = (x_2 - x_1)(a - b) = \Delta x(a - b) \)
- \( d \) is always integer

\[
\begin{align*}
&j + \frac{3}{2} \\
&j + \frac{1}{2} \\
&j - \frac{1}{2} \\
&i + \frac{1}{2} \\
&i - \frac{1}{2}
\end{align*}
\]

Bresenham's Algorithm III

- Compute d at step k + 1 from d at step k!
- Case: j did not change (\( d_k > 0 \))
  - a decreases by m, b increases by m
  - \((a - b)\) decreases by \(2m = 2(\Delta y/\Delta x)\)
  - \(\Delta x(a-b)\) decreases by \(2\Delta y\)

\[
\begin{align*}
&j + \frac{3}{2} \\
&j + \frac{1}{2} \\
&j - \frac{1}{2} \\
&i + \frac{1}{2} \\
&i - \frac{1}{2}
\end{align*}
\]

Bresenham's Algorithm IV

- Case: j did change (\( d_k \leq 0 \))
  - a decreases by m − 1, b increases by m − 1
  - \((a - b)\) decreases by \(2m - 2 = 2(\Delta y/\Delta x - 1)\)
  - \(\Delta x(a-b)\) decreases by \(2(\Delta y - \Delta x)\)

\[
\begin{align*}
&j + \frac{3}{2} \\
&j + \frac{1}{2} \\
&j - \frac{1}{2} \\
&i + \frac{1}{2} \\
&i - \frac{1}{2}
\end{align*}
\]

Bresenham's Algorithm V

- So \( d_{k+1} = d_k - 2\Delta y \) if \( d_k > 0 \)
- And \( d_{k+1} = d_k - 2(\Delta y - \Delta x) \) if \( d_k \leq 0 \)
- Final (efficient) implementation:
  ```c
  void draw_line(int x1, int y1, int x2, int y2) {
    int x, y = y1;
    int dx = 2*(x2-x1), dy = 2*(y2-y1);
    int dydx = dy-dx, D = (dy-dx)/2;
    for (x = x1 ; x <= x2 ; x++) {
      write_pixel(x, y, color);
      if (D > 0) D -= dy;
      else {y++; D -= dydx;
    }
  }
  
  ```

Bresenham's Algorithm VI

- Need different cases to handle \( m > 1 \)
- Highly efficient
- Easy to implement in hardware and software
- Widely used
Outline

- Scan Conversion for Lines
- Scan Conversion for Polygons
- Antialiasing

Scan Conversion of Polygons

- Multiple tasks:
  - Filling polygon (inside/outside)
  - Pixel shading (color interpolation)
  - Blending (accumulation, not just writing)
  - Depth values (z-buffer hidden-surface removal)
  - Texture coordinate interpolation (texture mapping)
- Hardware efficiency is critical
- Many algorithms for filling (inside/outside)
- Much fewer that handle all tasks well

Filling Convex Polygons

- Find top and bottom vertices
- List edges along left and right sides
- For each scan line from bottom to top
  - Find left and right endpoints of span, $x_l$ and $x_r$
  - Fill pixels between $x_l$ and $x_r$
  - Can use Bresenham’s algorithm to update $x_l$ and $x_r$

Concave Polygons: Odd-Even Test

- Approach 1: odd-even test
- For each scan line
  - Find all scan line/polygon intersections
  - Sort them left to right
  - Fill the interior spans between intersections
- Parity rule: inside after an odd number of crossings

Edge vs Scan Line Intersections

- Brute force: calculate intersections explicitly
- Incremental method (Bresenham’s algorithm)
- Caching intersection information
  - Edge table with edges sorted by $y_{\text{min}}$
  - Active edges, sorted by x-intersection, left to right
- Process image from smallest $y_{\text{min}}$ up

Concave Polygons: Tessellation

- Approach 2: divide non-convex, non-flat, or non-simple polygons into triangles
- OpenGL specification
  - Need accept only simple, flat, convex polygons
  - Tessellate explicitly with tessellator objects
  - Implicitly if you are lucky
- Most modern GPUs scan-convert only triangles
**Flood Fill**
- Draw outline of polygon
- Pick color seed
- Color surrounding pixels and recurse
- Must be able to test boundary and duplication
- More appropriate for drawing than rendering

**Outline**
- Scan Conversion for Lines
- Scan Conversion for Polygons
- Antialiasing

**Aliasing**
- Artifacts created during scan conversion
- Inevitable (going from continuous to discrete)
- Aliasing (name from digital signal processing): we sample a continues image at grid points
- Effect
  - Jagged edges
  - Moire patterns

**More Aliasing**
- Moire pattern from sandlotscience.com

**Antialiasing for Line Segments**
- Use area averaging at boundary
- (c) is aliased, magnified
- (d) is antialiased, magnified

**Antialiasing by Supersampling**
- Mostly for off-line rendering (e.g., ray tracing)
- Render, say, 3x3 grid of mini-pixels
- Average results using a filter
- Can be done adaptively
  - Stop if colors are similar
  - Subdivide at discontinuities
Supersampling Example

- Other improvements
  - Stochastic sampling: avoid sample position repetitions
  - Stratified sampling (jittering): perturb a regular grid of samples

Temporal Aliasing

- Sampling rate is frame rate (30 Hz for video)
- Example: spokes of wagon wheel in movies
- Solution: supersample in time and average
  - Fast-moving objects are blurred
  - Happens automatically with real hardware (photo and video cameras)
  - Exposure time is important (shutter speed)
  - Effect is called motion blur

Wagon Wheel Effect

Source: YouTube

Motion Blur Example

Achieve by stochastic sampling in time

T. Porter, Pixar, 1984
16 samples / pixel / timestep

Summary

- Scan Conversion for Polygons
  - Basic scan line algorithm
  - Convex vs concave
  - Odd-even rules, tessellation

- Antialiasing (spatial and temporal)
  - Area averaging
  - Supersampling
  - Stochastic sampling