Spatial Data Structures

- Data structures to store geometric information
- Sample applications
  - Collision detection
  - Location queries
  - Chemical simulations
  - Rendering
- Spatial data structures for ray tracing
  - Object-centric data structures (bounding volumes)
  - Space subdivision (grids, octrees, BSP trees)
  - Speed-up of 10x, 100x, or more

Ray Tracing Acceleration

- Faster intersections
  - Faster ray-object intersections
    - Object bounding volume
    - Efficient intersectors
  - Fewer ray-object intersections
    - Hierarchical bounding volumes (boxes, spheres)
    - Spatial data structures
    - Directional techniques
- Fewer rays
  - Adaptive tree-depth control
  - Stochastic sampling
- Generalized rays (beams, cones)

Bounding Volumes

- Wrap complex objects in simple ones
- Does ray intersect bounding box?
  - No: does not intersect enclosed objects
  - Yes: calculate intersection with enclosed objects
- Common types:
  - Sphere
  - Axis-aligned Bounding Box (AABB)
  - Oriented Bounding Box (OBB)
  - 6-dop
  - Convex Hull

Selection of Bounding Volumes

- Effectiveness depends on:
  - Probability that ray hits bounding volume, but not enclosed objects (tight fit is better)
  - Expense to calculate intersections with bounding volume and enclosed objects
- Amortize calculation of bounding volumes
- Use heuristics

Hierarchical Bounding Volumes

- With simple bounding volumes, ray casting still requires \( O(n) \) intersection tests
- Idea: use tree data structure
  - Larger bounding volumes contain smaller ones etc.
  - Sometimes naturally available (e.g. human figure)
  - Sometimes difficult to compute
- Often reduces complexity to \( O(\log(n)) \)
**Ray Intersection Algorithm**

- Recursively descend tree
- If ray misses bounding volume, no intersection
- If ray intersects bounding volume, recurse with enclosed volumes and objects
- Maintain near and far bounds to prune further
- Overall effectiveness depends on model and constructed hierarchy

**Spatial Subdivision**

- Bounding volumes enclose objects, recursively
- Alternatively, divide space (as opposed to objects)
- For each segment of space, keep a list of intersecting surfaces or objects
- Basic techniques:
  - Uniform Spatial Sub
  - Quadtree/Octree
  - kd-tree
  - BSP-tree

**Grids**

- 3D array of cells (voxels) that tile space
- Each cell points to all intersecting surfaces
- Intersection algorithm steps from cell to cell

**Caching Intersection Points**

- Objects can span multiple cells
- For A need to test intersection only once
- For B need to cache intersection and check next cell for any closer intersection with other objects
- If not, C could be missed (yellow ray)

**Assessment of Grids**

- Poor choice when world is non-homogeneous
- Size of grid
  - Too small: too many surfaces per cell
  - Too large: too many empty cells to traverse
  - Can use algorithms like Bresenham’s for efficient traversal
- Non-uniform spatial subdivision more flexible
  - Can adjust to objects that are present

**Outline**

- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- BSP Trees
**Quadtrees**
- Generalization of binary trees in 2D
  - Node (cell) is a square
  - Recursively split into 4 equal sub-squares
  - Stop subdivision based on number of objects
- Ray intersection has to traverse quadtree
- More difficult to step to next cell

**Octrees**
- Generalization of quadtree in 3D
  - Each cell may be split into 8 equal sub-cells
  - Internal nodes store pointers to children
  - Leaf nodes store list of surfaces
  - Adapts well to non-homogeneous scenes

**Assessment for Ray Tracing**
- Grids
  - Easy to implement
  - Require a lot of memory
  - Poor results for non-homogeneous scenes
- Octrees
  - Better on most scenes (more adaptive)
- Alternative: nested grids
- Spatial subdivision expensive for animations
- Hierarchical bounding volumes
  - Natural for hierarchical objects
  - Better for dynamic scenes

**Other Spatial Subdivision Techniques**
- Relax rules for quadtrees and octrees
- k-dimensional tree (k-d tree)
  - Split at arbitrary interior point
  - Split one dimension at a time
- Binary space partitioning tree (BSP tree)
  - In 2 dimensions, split with any line
  - In k dims, split with k-1 dimensional hyperplane
  - Particularly useful for painter's algorithm
  - Can also be used for ray tracing

**Outline**
- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- BSP Trees

**BSP Trees**
- Split space with any line (2D) or plane (3D)
- Applications
  - Painters algorithm for hidden surface removal
  - Ray casting
- Inherent spatial ordering given viewpoint
  - Left subtree: in front, right subtree: behind
- Problem: finding good space partitions
  - Proper ordering for any viewpoint
  - How to balance the tree
Building a BSP Tree
- Use hidden surface removal as intuition
- Using line 1 or line 2 as root is easy

Building a Good Tree
- Naive partitioning of \( n \) polygons yields \( O(n^3) \) polygons (in 3D)
- Algorithms with \( O(n^2) \) increase exist
  - Try all, use polygon with fewest splits
  - Do not need to split exactly along polygon planes
- Should balance tree
  - More splits allow easier balancing
  - Rebalancing?

Splitting of surfaces
- Using line 3 as root requires splitting

Painter’s Algorithm with BSP Trees
- Building the tree
  - May need to split some polygons
    - Slow, but done only once
- Traverse back-to-front or front-to-back
  - Order is viewer-direction dependent
  - What is front and what is back of each line changes
  - Determine order on the fly

Details of Painter’s Algorithm
- Each face has form \( Ax + By + Cz + D \)
- Plug in coordinates and determine
  - Positive: front side
  - Zero: on plane
  - Negative: back side
- Back-to-front: inorder traversal, farther child first
- Front-to-back: inorder traversal, near child first
- Do backface culling with same sign test
- Clip against visible portion of space (portals)

Clipping With Spatial Data Structures
- Accelerate clipping
  - Goal: accept or rejects whole sets of objects
  - Can use an spatial data structures
- Scene should be mostly fixed
  - Terrain fly-through
  - Gaming
  - Viewing frustum

Hierarchical bounding volumes
Octrees
Data Structure Demos

- BSP Tree construction
  http://symbolcraft.com/graphics/bsp/index.html

- KD Tree construction
  http://donar.umiacs.umd.edu/quadtree/points/kd-tree.html

Real-Time and Interactive Ray Tracing

- Interactive ray tracing via space subdivision
  http://www.cs.utah.edu/~reinhard/egwr/

- State of the art in interactive ray tracing
  http://www.cs.utah.edu/~shirley/irt/

Summary

- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- BSP Trees