CSCI 480 Computer Graphics
Lecture 21

Physically Based Simulation

Examples
Particle Systems
Numerical Integration
Cloth Simulation
[Ch. 11.2-11.6]

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Jernej Barbic
University of Southern California
http://www-bcf.usc.edu/~jbarbic/cs480-s11/

Physics in Computer Graphics

• Very common
• Computer Animation, Modeling
  (computational mechanics)
• Rendering (computational optics)

Physics in Computer Animation

• Fluids
• Smoke
• Deformable strands (rods)
• Cloth
• Solid 3D deformable objects .... and many more!

Fluids

Evright, Marschner, Fedkiw,
SIGGRAPH 2002

Fluids and Rigid Bodies

[Carlson, Mucha, Turk,
SIGGRAPH 2004]

Fluids with Deformable Solid Coupling

[Robinson-Mosher,
Shinar, Gretarsson,
Su, Fedkiw,
SIGGRAPH 2008]

Two-way Coupling of Fluids to Rigid and Deformable Solids and Shells
Avi Robinson-Mosher
Tamar Shinar
Jon Gretarsson
Jonathan Su
Ronald Fedkiw
Deformations

[Barbic and James, SIGGRAPH 2005]

Cloth

Source: ACM SIGGRAPH

Cloth (Robustness)

[Bridson, Fedkiw, Anderson, ACM SIGGRAPH 2002]

Simulating Large Models

[Doug James, PhD Thesis, UBC, 2001]

Sound Simulation (Acoustics)

[James, Barbic, Pai, SIGGRAPH 2006]

Multibody Dynamics

Figure 1: Avalanche: 300 rocks tumble down a mountainside.
Multibody Dynamics + Self-collision Detection

Physics in Games

Haptic Interfaces

Surgical Simulation

Offline Physics

Real-time Physics

Haptic Interfaces


• hap-tic ('hap-tik')
  adj.
  Of or relating to the sense of touch; tactile.

Physics in Games

Real-Time Deformation and Fracture in a Game Environment

Eric Parker
Pixelux Entertainment
James O'Brien
U.C. Berkeley

Video Edited by Sebastian Burke
From the proceedings of SCA 2009, New Orleans

Surgical Simulation

[James and Pai, SIGGRAPH 2002]

Offline Physics

• Special effects (film, commercials)
• Large models:
  millions of particles / tetrahedra / triangles
• Use computationally expensive rendering
  (global illumination)
• Impressive results
• Many seconds of computation time per frame

Real-time Physics

• Interactive systems:
  computer games
  virtual medicine (surgical simulation)
• Must be fast (30 fps, preferably 60 fps for games)
  Only a small fraction of CPU time devoted to physics!
• Has to be stable, regardless of user input

[Barbic and James, SIGGRAPH 2010]

[Parker and James, Symposium on Computer Animation 2009]
Particle System

- Basic physical system in computer graphics
- We have N particles
- They interact with some forces
- Fire, Smoke, Cloth, ...
- Very popular for its simplicity

[William Reeves, SIGGRAPH 1983]

Newton’s Laws

- Newton’s 2nd law:
  \[ \vec{F} = m \vec{a} \]
- Gives acceleration, given the force and mass
- Newton’s 3rd law: If object A exerts a force \( \vec{F} \) on object B, then object B is at the same time exerting force \(-\vec{F}\) on A.

Case Study: Mass-spring Systems

- Mass particles connected by elastic springs
- One dimensional: rope, chain
- Two dimensional: cloth, shells
- Three dimensional: soft bodies

Source: Matthias Mueller, SIGGRAPH

Single spring

- Obeys the Hook’s law:
  \[ F = k (x - x_0) \]
- \( x_0 \) = rest length
- \( k \) = spring elasticity (stiffness)
- For \( x < x_0 \), spring wants to extend
- For \( x > x_0 \), spring wants to contract

Hook’s law in 3D

- Assume A and B two mass points connected with a spring.
- Let \( L \) be the vector pointing from B to A
- Let \( R \) be the spring rest length
- Then, the elastic force exerted on A is:

\[ \vec{F} = -k_{\text{Hook}}(\vec{L} - \vec{R}) \frac{\vec{L}}{|\vec{L}|} \]

Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

\[ \vec{F} = -k_d \vec{v} \]

- \( k_d \) = damping coefficient
- \( k_d \) different than \( k_{\text{Hook}} \)!!
A network of springs

- Every mass point connected to some other points by springs
- Springs exert forces on mass points
  - Hooke’s force
  - Damping force
- Other forces
  - External force field
    - Gravity
    - Electrical or magnetic force field
  - Collision force

Network organization is critical

- For stability, must organize the network of springs in some clever way

Basic network
Stable network
Network out of control

Time Integration

Physics equation:

\[ x' = f(x, t) \]

\[ x = x(t) \] is particle trajectory

Euler Integration

\[ x(t + \Delta t) = x(t) + \Delta t \cdot f(x(t)) \]

Simple, but inaccurate.
Unstable with large timesteps.

Inaccuracies with explicit Euler

Gain energy
“Blow-up”

Midpoint Method

Improves stability

1. Compute Euler step
\[ \Delta x = \Delta t \cdot f(x, t) \]
2. Evaluate f at the midpoint
\[ f_{mid} = f((x+\Delta x)/2, (t+\Delta t)/2) \]
3. Take a step using the midpoint value
\[ x(t + \Delta t) = x(t) + \Delta t \cdot f_{mid} \]
Many more methods

- Runge-Kutta (4th order and higher orders)

- Implicit methods
  - sometimes unconditionally stable
  - very popular (e.g., cloth simulations)
  - a lot of damping with large timesteps

- Symplectic methods
  - exactly preserve energy, angular momentum and/or other physical quantities
  - Symplectic Euler

Cloth Simulation

- Cloth Forces
  - Stretch
  - Shear
  - Bend

- Many methods are a more advanced version of a mass-spring system
- Derivatives of Forces
  - necessary for stability

Challenges

- Complex Formulas
- Large Matrices
- Stability
- Collapsing triangles
- Self-collision detection

Self-collisions: definition

Deformable model is self-colliding iff there exist non-neighboring intersecting triangles.

Bounding volume hierarchies

AABBs Level 1
AABBs Level 3

Bounding volume hierarchy

root

[Baraff and Witkin, SIGGRAPH 1998]
[Govindaraju et al. 2005]
**Bounding volume hierarchy**

```
  V  W
```

**Real-time cloth simulation**

<table>
<thead>
<tr>
<th>Model</th>
<th>Triangles</th>
<th>FPS</th>
<th>% Forces + Stiffness Matrix</th>
<th>% Solver</th>
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<td>25</td>
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<td>33</td>
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</tbody>
</table>

**Multithreading implementation**

Source: Andy Pierce

**Summary**

- Examples of physically based simulation
- Particle Systems
- Numerical Integration
- Cloth Simulation