Hierarchical Models

Jernej Barbic
University of Southern California
Roadmap

• Last lecture: Viewing and projection
• Today:
  – Shadows via projections
  – Hierarchical models
• Next: Polygonal Meshes, Curves and Surfaces
• Goal: background for Assignment 2 (next week)
Importance of shadows

Source: UNC
Importance of shadows
Importance of shadows
Importance of shadows

Without shadows

With shadows

Source: UNC
Doom III

Reported to spend 50% of time rendering shadows!

Light sources

- Point light source
- Directional light source
- Area light source
Hard and soft shadows

Source: UNC
Shadow Algorithms

• With visibility tests
  – Accurate yet expensive
  – Example: ray casting or ray tracing
  – Example: 2-pass z-buffer
    [Foley, Ch. 16.4.4] [RTR 6.12]

• Without visibility tests (“fake” shadows)
  – Approximate and inexpensive
  – Using a model-view matrix “trick”
Projection-based Shadows

• Assume light source at \([x_l, y_l, z_l]^T\)
• Assume shadow on plane \(y = 0\)
• Viewing = shadow projection
  – Center of projection = light
  – Viewing plane = shadow plane
• Construct a modelview matrix to flatten the geometry onto the shadow plane
Shadow Projection Strategy

- Move light source to origin
- Apply appropriate projection matrix
- Move light source back
- Instance of general strategy: compose complex transformation from simpler ones!

\[ T = \begin{bmatrix} 1 & 0 & 0 & -x_l \\ 0 & 1 & 0 & -y_l \\ 0 & 0 & 1 & -z_l \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
Derive Equation

• Now, light source at origin

\[
\begin{align*}
    x_p &= \frac{x}{y} \\
    y_p &= \frac{y}{y} \quad \text{(see picture)} \\
    y_p &= -y_l \quad \text{(move light)} \\
    x_p &= \frac{x}{y}y_p = -\frac{x}{y}y_l \\
    z_p &= \frac{z}{y}y_p = -\frac{z}{y}y_l
\end{align*}
\]
Light Source at Origin

- After translation, solve

\[
M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = w \begin{bmatrix} - \frac{xy_l}{y} \\ - \frac{y_l}{zy_l} \\ - \frac{y}{y_l} \\ 1 \end{bmatrix}
\]

- \( w \) can be chosen freely
- Use \( w = -y / y_l \)
Shadow Projection Matrix

- Solution of previous equation

\[
M = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -\frac{1}{y_l} & 0 & 0 & 0
\end{bmatrix}
\]

- Total shadow projection matrix

\[
S = T^{-1}MT = \ldots
\]
Implementation

• Recall column-major form
  
  GLfloat m[16] =
  {1.0, 0.0, 0.0, 0.0,
   0.0, 1.0, 0.0, -1.0 / yl,
   0.0, 0.0, 1.0, 0.0,
   0.0, 0.0, 0.0, 0.0};

• yl is light source height
• Assume drawPolygon(); draws object
Saving the ModelView Matrix State

• Assume $x_l$, $y_l$, $z_l$ hold light coordinates
• Core OpenGL code (compatibility code is similar)

```cpp
goLMatrixMatrixMode(openGLMatrix::ModelView);
// here, set the model view matrix, in the usual way
// …

drawPolygon(); // draw normally
openGLMatrix.PushMatrix(); // save current matrix
openGLMatrix.Translate($x_l$, $y_l$, $z_l$); // translate back
openGLMatrix.MultMatrix($m$); // project
openGLMatrix.Translate(-$x_l$, -$y_l$, -$z_l$); // move light to origin

float ms[16];
openGLMatrix.GetMatrix(ms); // read the shadow matrix
```
Saving the ModelView Matrix State (cont.)

// upload the shadow matrix to the GPU
glUniformMatrix4fv(h_modelViewMatrix, 1, GL_FALSE, ms);

drawPolygon();  // draw polygon again for shadow

// restore original modelview matrix
openGLMatrix.PopMatrix();
openGLMatrix.GetMatrix(ms);
glUniformMatrix4fv(h_modelViewMatrix, 1, GL_FALSE, ms);

// continue rendering more objects, as usual …
The Matrix and Attribute Stacks

• Mechanism to save and restore state
  – {OpenGLMatrix::, gl}PushMatrix();
  – {OpenGLMatrix::, gl}PopMatrix();

• Apply to current matrix

• In compatibility profile, can also save current attribute values
  – Examples: color, lighting
  – glPushAttrib(GLbitfield mask);
  – glPopAttrib();
  – Mask determines which attributes are saved
  – This feature has been removed in the core profile
Drawing on a Surface

- Shimmering (“z-buffer fighting”) when drawing shadow on surface
- Due to limited precision of depth buffer
- Solution: slightly displace either the surface or the shadow

(glPolygonOffset in OpenGL)
Drawing on a Surface

Or use general technique

1. Set depth buffer to read-only, draw surface
2. Set depth buffer to read-write, draw shadow
3. Set color buffer to read-only, draw surface again
4. Set color buffer to read-write
Outline

• Projections and Shadows
• Hierarchical Models
Hierarchical Models

• Many graphical objects are structured

• Exploit structure for
  – Efficient rendering
  – Example: tree leaves
  – Concise specification of model parameters
  – Example: joint angles
  – Physical realism

• Structure often naturally hierarchical
Instance Transformation

- Often we need several instances of an object
  - Wheels of a car
  - Arms or legs of a figure
  - Chess pieces
Instance Transformation

- Instances can be shared across space or time
- Write a function that renders the object in “standard” configuration
- Apply transformations to different instances
- Typical order: scaling, rotation, translation
Sample Instance Transformation

```cpp
openGLMatrix.MatrixMode(OpenGLMatrix::ModelView);
openGLMatrix.LoadIdentity();
openGLMatrix.Translate(...);
openGLMatrix.Rotate(...);
openGLMatrix.Scale(...);
// … upload modelview matrix to GPU, as usual … renderCylinder(...);
```
Drawing a Compound Object

• Example: simple “robot arm”

Base rotation $\theta$, arm angle $\phi$, joint angle $\psi$
Hierarchical Objects and Animation

- Drawing functions are time-invariant and draw the object in a canonical position:
  
  ```cpp
drawBase(); drawLowerArm(); drawUpperArm();
  ```

- Can be easily stored in a VBO
- Change parameters of model with time
Interleave Drawing & Transformation

- $h1 =$ height of base, $h2 =$ length of lower arm
- This is pseudocode (must upload matrix to GPU)

```c
void drawRobot(GLfloat theta, GLfloat phi, GLfloat psi)
{
    Rotate(theta, 0.0, 1.0, 0.0);
    drawBase();
    Translate(0.0, h1, 0.0);
    Rotate(phi, 0.0, 0.0, 1.0);
    drawLowerArm();
    Translate(0.0, h2, 0.0);
    Rotate(psi, 0.0, 0.0, 1.0);
    drawUpperArm();
}
```
More Complex Objects

- Tree rather than linear structure
- Interleave along each branch
- Use push and pop to save state
Hierarchical Tree Traversal

- Order not necessarily fixed (breadth-first, depth-first, etc.)
- Example:

```cpp
void drawFigure()
{
    PushMatrix(); // save
    drawTorso();
    Translate(...); // move head
    Rotate(...);   // rotate head
    drawHead();
    PopMatrix();   // restore
    PushMatrix();
    Translate(...);
    Rotate(...);
    drawLeftUpperArm();
    Translate(...)
    Rotate(...)
    drawLeftLowerArm();
    PopMatrix();
    ...
}
```
Summary

• Projections and Shadows
• Hierarchical Models