Polygon Meshes and Implicit Surfaces

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Polygon Meshes
Implicit Surfaces
Constructive Solid Geometry
[Angel Ch. 10]
Modeling Complex Shapes

• An equation for a sphere is possible, but how about an equation for a telephone, or a face?

• Complexity is achieved using simple pieces
  – polygons, parametric surfaces, or implicit surfaces

• Goals
  – Model anything with arbitrary precision (in principle)
  – Easy to build and modify
  – Efficient computations (for rendering, collisions, etc.)
  – Easy to implement (a minor consideration...)

What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

No single technique solves all problems!
Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces
Polygon Meshes

• Any shape can be modeled out of polygons
  – if you use enough of them…

• Polygons with how many sides?
  – Can use triangles, quadrilaterals, pentagons, … n-gons
  – Triangles are most common.
  – When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting.

• Polygon meshes are built out of
  – *vertices* (points)
  – *edges* (line segments between vertices)
  – *faces* (polygons bounded by edges)
Normals

Triangle defines unique plane
- can easily compute normal
  \[ \mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{||\mathbf{a} \times \mathbf{b}||} \]
- depends on vertex orientation!
- clockwise order gives
  \[ \mathbf{n}' = -\mathbf{n} \]

Vertex normals less well defined
- can average face normals
- works for smooth surfaces
- but not at sharp corners
  - think of a cube
Where Meshes Come From

• Specify manually
  – Write out all polygons
  – Write some code to generate them
  – Interactive editing: move vertices in space

• Acquisition from real objects
  – Laser scanners, vision systems
  – Generate set of points on the surface
  – Need to convert to polygons
Data Structures for Polygon Meshes

• Simplest (but dumb)
  – float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
  – redundant: each vertex stored multiple times

• Vertex List, Face List
  – List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
  – List of triangles, each a triple of vertex id’s (or pointers) topological (connectivity, adjacency) info only

  Fine for many purposes, but finding the faces adjacent to a vertex takes \( O(F) \) time for a model with \( F \) faces. Such queries are important for topological editing.

• Fancier schemes:
  Store more topological info so adjacency queries can be answered in \( O(1) \) time.

  Winged-edge data structure – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).
A File Format for Polygon Models: OBJ

# OBJ file for a 2x2x2 cube
v -1.0 1.0 1.0  - vertex 1
v -1.0 -1.0 1.0  - vertex 2
v 1.0 -1.0 1.0  - vertex 3
v 1.0 1.0 1.0
v -1.0 1.0 -1.0
v -1.0 -1.0 -1.0
v 1.0 -1.0 -1.0
v 1.0 1.0 -1.0
v 1.0 1.0 -1.0
f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3

Syntax:

v x y z  - a vertex at (x,y,z)

f v1 v2 ... vn  - a face with vertices v1, v2, ... vn

# anything  - comment
How Many Polygons to Use?

5802 triangles

800 triangles

300 triangles

100 triangles
Why Level of Detail?

• Different models for near and far objects
• Different models for rendering and collision detection
• Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without
• losing key features
• getting artifacts in the silhouette
• popping
Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes
- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?
Shape Representations

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Parametric Surfaces

\[ p(u,v) = [x(u,v), y(u,v), z(u,v)] \]

- e.g. plane, cylinder, bicubic surface, swept surface

Beziers patch
Parametric Surfaces

\[ p(u,v) = [x(u,v), y(u,v), z(u,v)] \]

- e.g. plane, cylinder, bicubic surface, swept surface

the Utah teapot
Parametric Surfaces

Why better than polygon meshes?

- Much more compact
- More convenient to control --- just edit control points
- Easy to construct from control points

What are the problems?

- Work well for smooth surfaces
- Must still split surfaces into discrete number of patches
- Rendering times are higher than for polygons
- Intersection test? Inside/outside test?
Shape Representations

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Two Ways to Define a Circle

**Parametric**

$x = f(u) = r \cos(u)$

$y = g(u) = r \sin(u)$

**Implicit**

$F(x, y) = x^2 + y^2 - r^2$

$F > 0$

$F = 0$

$F < 0$
Implicit Surfaces

- Implicit surface: \( F(x,y,z) = 0 \)
  - e.g. plane, sphere, cylinder, quadric, torus, blobby models
  - sphere with radius \( r \): \( F(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0 \)
  - terrible for iterating over the surface
  - great for intersections, inside/outside test

- well defined inside/outside
- polygons and parametric surfaces do not have this information

- Computing is hard:
  implicit functions for a cube? telephone?

\[ F(x,y) = x^2 + y^2 - r^2 = 0 \]
Quadric Surfaces

\[ F(x,y,z) = ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2px+2qy+2rz+d = 0 \]
What Implicit Functions are Good For

Ray - Surface Intersection Test

Inside/Outside Test
Surfaces from Implicit Functions

• Constant Value Surfaces are called (depending on whom you ask):
  – constant value surfaces
  – level sets
  – isosurfaces

• Nice Feature: you can add them! (and other tricks)
  – this merges the shapes
  – When you use this with spherical exponential potentials, it’s called *Blobs, Metaballs, or Soft Objects*. Great for modeling animals.
Blobby Models

by Brian Wyvill, http://www.cpsc.ucalgary.ca/~blob/
How to draw implicit surfaces?

• It’s easy to ray trace implicit surfaces
  – because of that easy intersection test
• Volume Rendering can display them
• Convert to polygons: the Marching Cubes algorithm
  – Divide space into cubes
  – Evaluate implicit function at each cube vertex
  – Do root finding or linear interpolation along each edge
  – Polygonize on a cube-by-cube basis
Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together
Constructive Solid Geometry (CSG)

**union**  
the merger of two objects into one

**difference**  
the subtraction of one object from another

**intersection**  
the portion common to both objects
Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together

- This is sensible for objects that are actually made that way (human-made, particularly machined objects)
A CSG Train

Brian Wyvill & students, Univ. of Calgary
Negative Objects

Use point-by-point boolean functions
- remove a volume by using a negative object
- e.g. drill a hole by subtracting a cylinder

\[
\text{Inside(BLOCK-CYL)} = \text{Inside(BLOCK)} \land \neg \text{Inside(CYL)}
\]
Set Operations

• UNION: \( \text{Inside}(A) \lor \text{Inside}(B) \)
  \[ \Rightarrow \text{Join A and B} \]

• INTERSECTION: \( \text{Inside}(A) \land \text{Inside}(B) \)
  \[ \Rightarrow \text{Chop off any part of A that sticks out of B} \]

• SUBTRACTION: \( \text{Inside}(A) \land \neg \text{Inside}(B) \)
  \[ \Rightarrow \text{Use B to Cut A} \]

Examples:
- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.
Implicit Functions for Booleans

• Recall the implicit function for a solid: \( F(x,y,z) < 0 \)

• Boolean operations are replaced by arithmetic:
  - \( \text{MAX} \) replaces \( \text{AND} \) (intersection)
  - \( \text{MIN} \) replaces \( \text{OR} \) (union)
  - \( \text{MINUS} \) replaces \( \text{NOT} \) (unary subtraction)

• Thus
  - \( F(\text{Intersect}(A,B)) = \text{MAX}(F(A), F(B)) \)
  - \( F(\text{Union}(A,B)) = \text{MIN}(F(A), F(B)) \)
  - \( F(\text{Subtract}(A,B)) = \text{MAX}(F(A), -F(B)) \)
CSG Trees

- Set operations yield tree-based representation

Implicit Surfaces

- Good for smoothly blending multiple components
- Clearly defined solid along with its boundary
- Intersection test and Inside/outside test are easy

- Need to polygonize to render --- expensive
- Interactive control is not easy
- Fitting to real world data is not easy
- Always smooth
Summary

- Polygonal Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry