Clipping

Line Clipping
Polygon Clipping
Clipping in Three Dimensions
[Angel Ch. 7.1-7.7]
The Graphics Pipeline, Revisited

- Must eliminate objects that are outside of viewing frustum
- **Clipping**: object space (eye coordinates)
- **Scissoring**: image space (pixels in frame buffer)
  - most often less efficient than clipping
- We will first discuss **2D clipping** (for simplicity)
  - OpenGL uses 3D clipping
Clipping Against a Frustum

- General case of frustum (truncated pyramid)

- Clipping is tricky because of frustum shape
Perspective Normalization

• Solution:
  – Implement perspective projection by **perspective normalization** and orthographic projection
  – Perspective normalization is a homogeneous transformation

See [Angel Ch. 5.9]
The Normalized Frustum

• OpenGL uses \(-1 \leq x, y, z \leq 1\) (others possible)

• Clip against resulting cube

• Clipping against arbitrary (programmer-specified) planes requires more general algorithms and is more expensive
The Viewport Transformation

• Transformation sequence again:
  1. **Camera**: From object coordinates to eye coords
  2. **Perspective normalization**: to clip coordinates
  3. **Clipping**
  4. **Perspective division**: to normalized device coords.
  5. **Orthographic projection** (setting $z_p = 0$)
  6. **Viewport transformation**: to screen coordinates

• Viewport transformation can distort
  – Solution: pass the correct window aspect ratio to `gluPerspective`
Clipping

- General: 3D object against cube

- Simpler case:
  - In 2D: line against square or rectangle
  - Later: polygon clipping
Clipping Against Rectangle in 2D

- **Line-segment clipping**: modify endpoints of lines to lie within clipping rectangle
Clipping Against Rectangle in 2D

- The result (in red)
Clipping Against Rectangle in 2D

- Could calculate intersections of line segments with clipping rectangle
  - expensive, due to floating point multiplications and divisions
- Want to minimize the number of multiplications and divisions

\[ y = k \cdot x + n \]

\[ x = x_0 \]

\[ y = y_0 \]

\[ x = x_1 \]

\[ y = y_1 \]
Several practical algorithms for clipping

- Main motivation:
  Avoid expensive line-rectangle intersections (which require floating point divisions)

- Cohen-Sutherland Clipping
- Liang-Barsky Clipping
- There are many more (but many only work in 2D)
Cohen-Sutherland Clipping

- Clipping rectangle is an intersection of 4 half-planes

- Encode results of four half-plane tests

- Generalizes to 3 dimensions (6 half-planes)
Outcodes (Cohen-Sutherland)

- Divide space into 9 regions
- 4-bit **outcode** determined by comparisons

\[
\begin{align*}
b_0 &: y > y_{\text{max}} \\
b_1 &: y < y_{\text{min}} \\
b_2 &: x > x_{\text{max}} \\
b_3 &: x < x_{\text{min}} \\
o_1 &= \text{outcode}(x_1, y_1) \\
o_2 &= \text{outcode}(x_2, y_2)
\end{align*}
\]
Cases for Outcodes

• Outcomes: accept, reject, subdivide

\[
o_1 = o_2 = 0000: \text{ accept entire segment}
\]

\[
o_1 \& o_2 \neq 0000: \text{ reject entire segment}
\]

\[
o_1 = 0000, o_2 \neq 0000: \text{ subdivide}
\]

\[
o_1 \neq 0000, o_2 = 0000: \text{ subdivide}
\]

\[
o_1 \& o_2 = 0000: \text{ subdivide}
\]
Cohen-Sutherland Subdivision

- Pick outside endpoint \( (o \neq 0000) \)
- Pick a crossed edge \( (o = b_0b_1b_2b_3 \text{ and } b_k \neq 0) \)
- Compute intersection of this line and this edge
- Replace endpoint with intersection point
- Restart with new line segment
  - Outcodes of second point are unchanged
- This algorithms converges
Liang-Barsky Clipping

- Start with parametric form for a line

\[ p(\alpha) = (1 - \alpha)p_1 + \alpha p_2, \quad 0 \leq \alpha \leq 1 \]
\[ x(\alpha) = (1 - \alpha)x_1 + \alpha x_2 \]
\[ y(\alpha) = (1 - \alpha)y_1 + \alpha y_2 \]
Liang-Barsky Clipping

- Compute all four intersections 1, 2, 3, 4 with extended clipping rectangle.
- Often, no need to compute all four intersections.
Ordering of intersection points

- Order the intersection points
- Figure (a): $1 > \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1 > 0$
- Figure (b): $1 > \alpha_4 > \alpha_2 > \alpha_3 > \alpha_1 > 0$
Liang-Barsky Idea

- It is possible to clip already if one knows the order of the four intersection points!
- Even if the actual intersections were not computed!
- Can enumerate all ordering cases
Liang-Barsky efficiency improvements

• Efficiency improvement 1:
  – Compute intersections one by one
  – Often can reject before all four are computed

• Efficiency improvement 2:
  – Equations for $\alpha_3$, $\alpha_2$

\[
\begin{align*}
  y_{\text{max}} &= (1 - \alpha_3)y_1 + \alpha_3 y_2 \\
  x_{\text{min}} &= (1 - \alpha_2)x_1 + \alpha_2 x_2 \\
  \alpha_3 &= \frac{y_{\text{max}} - y_1}{y_2 - y_1} \\
  \alpha_2 &= \frac{x_{\text{min}} - x_1}{x_2 - x_1}
\end{align*}
\]

  – Compare $\alpha_3$, $\alpha_2$ without floating-point division
Line-Segment Clipping Assessment

• Cohen-Sutherland
  – Works well if many lines can be rejected early
  – Recursive structure (multiple subdivisions) is a drawback

• Liang-Barsky
  – Avoids recursive calls
  – Many cases to consider (tedious, but not expensive)
Outline

• Line-Segment Clipping
  – Cohen-Sutherland
  – Liang-Barsky

• Polygon Clipping
  – Sutherland-Hodgeman

• Clipping in Three Dimensions
Polygon Clipping

- Convert a polygon into **one ore more** polygons
- Their union is intersection with clip window
- Alternatively, we can first tesselate concave polygons (OpenGL supported)
Concave Polygons

• Approach 1: clip, and then join pieces to a single polygon
  – often difficult to manage

• Approach 2: tesselate and clip triangles
  – this is the common solution
Sutherland-Hodgeman (part 1)

• Subproblem:
  – Input: polygon (vertex list) and single clip plane
  – Output: new (clipped) polygon (vertex list)

• Apply once for each clip plane
  – 4 in two dimensions
  – 6 in three dimensions
  – Can arrange in pipeline
Sutherland-Hodgeman (part 2)

• To clip vertex list (polygon) against a half-plane:
  – Test first vertex. Output if inside, otherwise skip.
  – Then loop through list, testing transitions
    • In-to-in: output vertex
    • In-to-out: output intersection
    • out-to-in: output intersection and vertex
    • out-to-out: no output
  – Will output clipped polygon as vertex list

• May need some cleanup in concave case
• Can combine with Liang-Barsky idea
Other Cases and Optimizations

• Curves and surfaces
  – Do it analytically if possible
  – Otherwise, approximate curves / surfaces by lines and polygons

• Bounding boxes
  – Easy to calculate and maintain
  – Sometimes big savings
Outline

• Line-Segment Clipping
  – Cohen-Sutherland
  – Liang-Barsky
• Polygon Clipping
  – Sutherland-Hodgeman
• Clipping in Three Dimensions
Clipping Against Cube

- Derived from earlier algorithms
- Can allow right parallelepiped
Cohen-Sutherland in 3D

- Use 6 bits in outcode
  - $b_4$: $z > z_{\text{max}}$
  - $b_5$: $z < z_{\text{min}}$
- Other calculations as before
Liang-Barsky in 3D

- Add equation $z(\alpha) = (1 - \alpha) z_1 + \alpha z_2$
- Solve, for $p_0$ in plane and normal $n$:
  
  $p(\alpha) = (1 - \alpha)p_1 + \alpha p_2$
  
  $n \cdot (p(\alpha) - p_0) = 0$

- Yields

  $\alpha = \frac{n \cdot (p_0 - p_1)}{n \cdot (p_2 - p_1)}$

- Optimizations as for Liang-Barsky in 2D
Summary: Clipping

- Clipping line segments to rectangle or cube
  - Avoid expensive multiplications and divisions
  - Cohen-Sutherland or Liang-Barsky

- Polygon clipping
  - Sutherland-Hodgeman pipeline

- Clipping in 3D
  - Essentially extensions of 2D algorithms
Preview and Announcements

• Scan conversion
• Anti-aliasing
• Other pixel-level operations
• Assignment 2 due a week from today!