CSCI 420 Computer Graphics
Lecture 7

Polygon Meshes and Implicit Surfaces

Polygon Meshes
Implicit Surfaces
Constructive Solid Geometry
[Angel Ch. 12.1-12.3]

Jernej Barbic
University of Southern California
Modeling Complex Shapes

- An equation for a sphere is possible, but how about an equation for a telephone, or a face?

- Complexity is achieved using simple pieces
  - polygons, parametric surfaces, or implicit surfaces

- Goals
  - Model anything with arbitrary precision (in principle)
  - Easy to build and modify
  - Efficient computations (for rendering, collisions, etc.)
  - Easy to implement (a minor consideration...)
What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

No single technique solves all problems!
Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces
Polygon Meshes

• Any shape can be modeled out of polygons
  – if you use enough of them…

• Polygons with how many sides?
  – Can use triangles, quadrilaterals,
    pentagons, … n-gons
  – Triangles are most common.
  – When > 3 sides are used, ambiguity about what to do
    when polygon nonplanar, or concave, or self-intersecting.

• Polygon meshes are built out of
  – vertices (points)
  – edges (line segments between vertices)
  – faces (polygons bounded by edges)
Polygon Models in OpenGL

• for faceted shading
  
glNormal3fv(n);
glBegin(GL_POLYGONS);
glVertex3fv(vert1);
glVertex3fv(vert2);
glVertex3fv(vert3);
glEnd();

• for smooth shading
  
glBegin(GL_POLYGONS);
glNormal3fv(normal1);
glVertex3fv(vert1);
glVertex3fv(vert2);
glVertex3fv(vert3);
glEnd();
Normals

**Triangle defines unique plane**
- can easily compute normal
  \[ \mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{\left| \mathbf{a} \times \mathbf{b} \right|} \]
- depends on vertex orientation!
- clockwise order gives
  \[ \mathbf{n}' = -\mathbf{n} \]

**Vertex normals less well defined**
- can average face normals
- works for smooth surfaces
- but not at sharp corners
  - think of a cube
Where Meshes Come From

- **Specify manually**
  - Write out all polygons
  - Write some code to generate them
  - Interactive editing: move vertices in space

- **Acquisition from real objects**
  - Laser scanners, vision systems
  - Generate set of points on the surface
  - Need to convert to polygons
Data Structures for Polygon Meshes

- Simplest (but dumb)
  - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
  - redundant: each vertex stored multiple times

- Vertex List, Face List
  - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
  - List of triangles, each a triple of vertex id’s (or pointers) topological (connectivity, adjacency) info only

  *Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.*

- Fancier schemes:
  Store more topological info so adjacency queries can be answered in O(1) time.

  *Winged-edge data structure* – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).
A File Format for Polygon Models: OBJ

# OBJ file for a 2x2x2 cube
v -1.0 1.0 1.0 - vertex 1
v -1.0 -1.0 1.0 - vertex 2
v 1.0 -1.0 1.0 - vertex 3
v 1.0 1.0 1.0
v -1.0 1.0 -1.0
v -1.0 -1.0 -1.0
v 1.0 -1.0 -1.0
v 1.0 1.0 -1.0
v 1.0 1.0 -1.0

f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3

Syntax:

v x y z - a vertex at (x,y,z)

f v₁ v₂ … vₙ - a face with vertices v₁, v₂, … vₙ

# anything - comment
How Many Polygons to Use?

5802 triangles
800 triangles
300 triangles
100 triangles
Why Level of Detail?

• Different models for near and far objects
• Different models for rendering and collision detection
• Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without
• losing key features
• getting artifacts in the silhouette
• popping
Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes
- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?
Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces
Parametric Surfaces

\[ p(u,v) = [x(u,v), y(u,v), z(u,v)] \]

- e.g. plane, cylinder, bicubic surface, swept surface
Parametric Surfaces

\[ p(u,v) = [x(u,v), y(u,v), z(u,v)] \]

- e.g. plane, cylinder, bicubic surface, swept surface

the Utah teapot
Parametric Surfaces

Why better than polygon meshes?

- Much more compact
- More convenient to control --- just edit control points
- Easy to construct from control points

What are the problems?

- Work well for smooth surfaces
- Must still split surfaces into discrete number of patches
- Rendering times are higher than for polygons
- Intersection test? Inside/outside test?
Shape Representations

Polygon Meshes
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Two Ways to Define a Circle

**Parametric**

\[ x = f(u) = r \cos(u) \]
\[ y = g(u) = r \sin(u) \]

**Implicit**

\[ F(x, y) = x^2 + y^2 - r^2 \]
Implicit Surfaces

- Implicit surface: $F(x,y,z) = 0$
  - e.g. plane, sphere, cylinder, quadric, torus, blobby models
  
  sphere with radius $r$: $F(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0$

- terrible for iterating over the surface
- great for intersections, inside/outside test
Quadric Surfaces

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0$$
What Implicit Functions are Good For

Ray - Surface Intersection Test

Inside/Outside Test
Surfaces from Implicit Functions

• Constant Value Surfaces are called (depending on whom you ask):
  – constant value surfaces
  – level sets
  – isosurfaces

• Nice Feature: you can add them! (and other tricks)
  – this merges the shapes
  – When you use this with spherical exponential potentials, it’s called *Blobs*, *Metaballs*, or *Soft Objects*. Great for modeling animals.
Blobby Models

by Brian Wyvill, http://www.cpsc.ucalgary.ca/~blob/
How to draw implicit surfaces?

• It’s easy to ray trace implicit surfaces
  – because of that easy intersection test
• Volume Rendering can display them
• Convert to polygons: the Marching Cubes algorithm
  – Divide space into cubes
  – Evaluate implicit function at each cube vertex
  – Do root finding or linear interpolation along each edge
  – Polygonize on a cube-by-cube basis
Constructive Solid Geometry (CSG)

• Generate complex shapes with basic building blocks
• Machine an object - saw parts off, drill holes, glue pieces together
Constructive Solid Geometry (CSG)

union
the merger of two objects into one

difference
the subtraction of one object from another

intersection
the portion common to both objects

Constructive Solid Geometry (CSG)

• Generate complex shapes with basic building blocks
• Machine an object - saw parts off, drill holes, glue pieces together
• This is sensible for objects that are actually made that way (human-made, particularly machined objects)
A CSG Train

Brian Wyvill & students, Univ. of Calgary
Negative Objects

Use point-by-point boolean functions

- remove a volume by using a negative object
- e.g. drill a hole by subtracting a cylinder

Inside(BLOCK-CYL) = Inside(BLOCK) And Not(Inside(CYL))
Set Operations

• UNION: \( \text{Inside}(A) \lor \text{Inside}(B) \)
  ➢ Join A and B

• INTERSECTION: \( \text{Inside}(A) \land \text{Inside}(B) \)
  ➢ Chop off any part of A that sticks out of B

• SUBTRACTION: \( \text{Inside}(A) \land \lnot \text{Inside}(B) \)
  ➢ Use B to Cut A

Examples:
- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.
Implicit Functions for Booleans

• Recall the implicit function for a solid: $F(x,y,z)<0$

• Boolean operations are replaced by arithmetic:
  - MAX replaces AND (intersection)
  - MIN replaces OR (union)
  - MINUS replaces NOT (unary subtraction)

• Thus
  - $F(\text{Intersect}(A,B)) = \text{MAX}(F(A), F(B))$
  - $F(\text{Union}(A,B)) = \text{MIN}(F(A), F(B))$
  - $F(\text{Subtract}(A,B)) = \text{MAX}(F(A), -F(B))$
CSG Trees

• Set operations yield tree-based representation
Implicit Surfaces

- Good for smoothly blending multiple components
- Clearly defined solid along with its boundary
- Intersection test and Inside/outside test are easy

- Need to polygonize to render --- expensive
- Interactive control is not easy
- Fitting to real world data is not easy
- Always smooth
Summary

- Polygonal Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry